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TURBULENT DISPERSION IN PULP FLOW: PRELIMINARY RESULTS AND IMPLICATIONS FOR THE MECHANISMS OF FIBER-TURBULENCE INTERACTIONS

C.O. LUETTGEN, J.D. LINDSAY, AND R.A. STRATTON

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Turbulent Dispersion in Pulp Flow: Preliminary Results and Implications for the Mechanisms of Fiber-Turbulence Interactions

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The Institute of Paper Science and Technology

Atlanta, GA 30318

ABSTRACT

A study of the kinetics of polymer adsorption and the influence of mixing conditions on adsorption homogeneity has been recently completed. As part of this study, experiments were performed on a flow loop entailing a side-port injection of a polymer/saline solution into turbulent flows of a dilute fiber suspension or of pure water. The influence of pipe and jet velocity conditions on turbulent dispersion were examined. Dispersion was quantified by cross-sectional conductivity measurements along the injection axis and at a right angle to the injection axis at various downstream positions. Although the data set provides information for only a small part of the flow, the data consistently show greater turbulent dispersion in the suspension flow than in a pure water flow, even though the suspension flow was apparently drag reducing (lower friction factor than for pure water alone). This counter-intuitive result can be related to some other studies of complex turbulence phenomena in suspension flows. The possibility of enhanced mass transfer in a drag reducing flow may be explained in terms of a proposed mechanism of floc-turbulence interaction which results in a restructuring of turbulence. The findings, though preliminary, justify further research in this area.

INTRODUCTION

Many techniques are used for introducing a polymeric retention aid to a pulp furnish. The mixing conditions during this dispersion process may play a role in the uniformity of the polymer adsorption and thus the efficiency of the retention aid addition. An investigation of the relationship between mixing conditions during polymer introduction, polymer adsorption uniformity and retention efficiency has been undertaken. A flow loop has been employed to examine the adsorption characteristics of a material injected into a turbulent flow of a fibrous suspension. As a side issue, some interesting results were found concerning turbulent dispersion when examining the influence of side-port jet and pipeline velocity conditions and the presence of fibers in the pipe. The dispersion results, and their relation to the theory of fiber-turbulence interactions, will be the focus of this report. We will first review several aspects of fiber suspension flow and other particle-fluid flows, including the phenomenon of drag reduction (polymeric drag reduction will also be discussed).

Flows of Fiber Suspensions

The unique characteristics of pulp slurries in pipe flow have been reported by many authors (1-9), and will only be touched upon here, loosely following Duffy et al. (2). Figure 1 shows a typical logarithmic head loss-velocity curve for a low consistency (say, fiber weight fraction < 6%) pulp suspension. In the region from A to B, plug flow of the fibrous network occurs. Near or slightly beyond B, a clear annulus of water with laminar flow may form around the plug; the annulus tends to be thin, typically less than a fiber length. Near C, turbulence in the annulus is apparent, with the fibers still forming a plug. The plug may be disrupted and begin to shrink at some point between C and E.

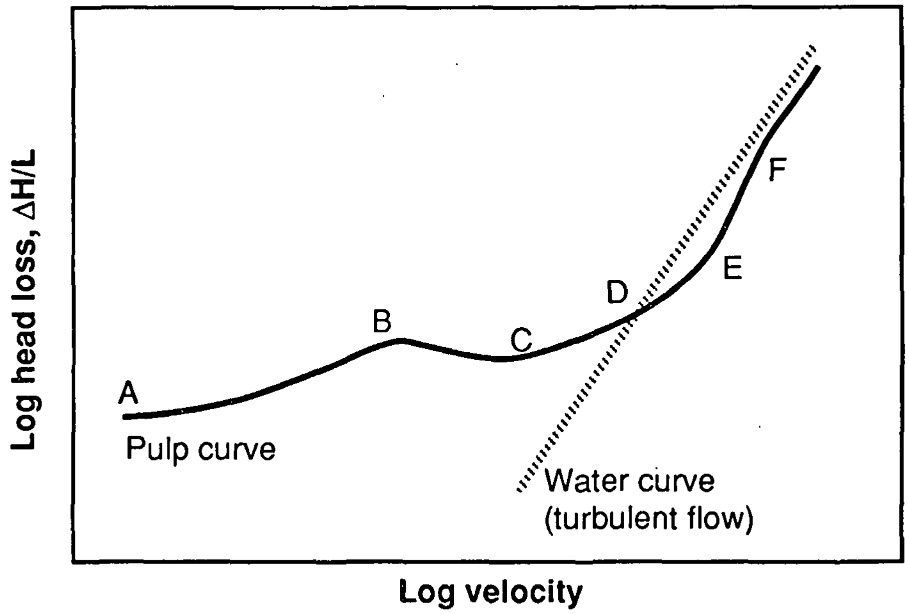

Figure 1. Comparison of head loss curves for water and a pulp suspension.

At point D, the pressure drop in the suspension is the same as in pure water at the same liquid flow rate. This marks the onset of drag reduction, for at all higher velocities the friction losses are less than for pure water, in spite of the higher apparent viscosity of the suspension. The degree of drag reduction, DR, is defined as

$$DR = \frac{(\Delta H/L)_{fluid+additive}}{(\Delta H/L)_{fluid}}, \; \bar{V}_{fluid} = \text{constant} \tag{1}$$

where $\Delta H/L$ is the head loss or pressure drop per unit length, and $\bar{V}_{fluid}$ is the average pure fluid velocity. The point of maximum drag reduction occurs at point E. Increases in velocity continue to disrupt the plug until the flow is fully turbulent, perhaps at point F. Drag reduction still occurs, although the degree of drag reduction tends to decrease as velocity increases further.

Details of the the head loss curve for pulp suspensions can vary widely depending on fiber properties, slurry concentration, and even configuration of the flow loop used in the measurements.

Drag Reduction

A discussion of drag reducing behavior in turbulent flows of fibers will assist in understanding some ways in which fibers modify turbulent structure. Before discussing drag reduction in fiber flows, however, we must mention the related phenomenon of drag reduction in polymer solutions.

Polymer Solutions

A variety of chemical additives have been found to lower friction losses in a turbulent fluid. The best known drag reducing agents are long, randomly coiled polymers which can decrease turbulent drag at concentrations as low as a few parts per million. Other additives for which drag reduction has been reported include soap micelles in solution (10). These flows typically exhibit increased longitudinal turbulent intensities, decreased radial turbulent intensity, increased time between turbulent bursts, and a reduction of energy at small scales, leading to less dissipation and longer-lived large eddies (11-13). Contradictory results exist, however, for in some cases the longitudinal turbulent intensity can either be increased or decreased during drag reduction (14). The drag reduction mechanism lies in the buffer region near the wall and depends only on the polymer concentration in that thin annular region (12).

A reasonable theory for the behavior of polymeric solutions has been offered by Lumley (15), who draws from the findings of many others. He suggests that the long polymer molecules, normally tightly coiled, become elongated when exposed to a sufficiently strong elongational strain. This cannot happen in the viscous sublayer, where the strain rate equals the vorticity: the molecules tend to be rotated instead of stretched. In the buffer layer, however, elongation can occur, and as the molecule uncoils, the effective viscosity of the solution increases dramatically and induces dampening of small-scale eddies which contain Reynolds stresses. Larger eddies, he suggests, will then be favored, and these will contribute to the large longitudinal fluctuations seen in most studies.

Fibrous Suspensions

In a study of drag reduction in suspensions of rigid nylon fibers, Kerekes and Douglas (16) observed drag reduction values as great as 50%. Drag reduction was found to be possible when the concentration was great enough for fiber-fiber interaction to occur, but below a critical concentration at which suspension viscosity dramatically increased. A critical parameter is the aspect ratio of the fiber, which must be greater than roughly 10-20 for drag reduction to be possible.

Vaseleski and Metzner (17) obtained flow resistance data for fiber suspensions in pipes with different diameters and inferred that the presence of fibers in the turbulent core region of flow was important for drag reduction. Likewise, Lee and Duffy (1), as well as Seely (18), found that turbulent velocity profiles in fibrous suspensions, when plotted in dimensionless turbulence coordinates, could be extrapolated to intersect at the point typically given as the end of the buffer layer in Newtonian fluids, although the profiles in the turbulent core had semi-logarithmic slopes lower than that of Newtonian fluids. This result was taken to indicate that the changes in turbulence mechanisms induced by fibers occur in the turbulent core, not at or near the wall. This is in contrast to drag reducing polymer solutions, in which the mechanism of drag reduction has been shown to occur near the wall (12). This difference may be simply due to the fact that fibers tend to migrate away from the wall and are not generally present there.

Unlike polymeric flows, drag reduction with fibers does not show a dependence on system scale (pipe diameter) (19). For example, Lee et al. (20) found drag reduction with a fiber additive alone remained approximately 20% as diameter increased from 2.5 to 7.0 cm at Re = 10^5. Some diameter dependence may be expected, however, if fiber migration away from the wall plays a significant role. (This is one reason why small tubes should not be used in studies of fibrous suspension flows if the results are to be applied to industrial applications.) Indeed, several authors have shown that the clear annulus that can form in fiber suspensions explains a number of important observations (21,22). Lumley (15), in fact, states that migration of large particles away from the wall can be an important mechanism in drag reduction. Fibers away from the wall will cause an increase in the effective viscosity in the turbulent core, damping turbulent fluctuations, while the viscosity in the viscous sublayer will be lower. This mechanism, however, may not account for the large degree of drag reduction observed in some dilutes suspensions where the effective viscosity of the suspension is nearly the same as that of the carrier fluid alone.

Radin et al. (21) conducted an extensive study of drag reduction in suspensions of fibrous and nonfibrous solids. Fibrous suspensions included chopped nylon and rayon fibers, asbestos, cotton fibers and reslushed newsprint. Nonfibrous solids included spherical, platelike, or needle-shaped rigid particles. No drag reduction was obtained with nonfibrous particles. They noted that some claims to the contrary are due to incorrect definitions of drag reduction in which authors compared friction factors in suspensions to those in a fluid with a similar average density. Properly defined, however, drag reduction means a lower friction factor than would occur in the solvent alone at the same solvent flow rate.

Radin et al. found that drag reduction could always be obtained in fibrous suspensions with L/D ratios greater than about 25-35. Key findings include:

- High L/D increases drag reduction.
- The degree of fiber dispersion strongly affects drag reduction for asbestos suspensions.
- Asbestos and paper fibers gave better drag reduction than nylon and rayon. While this is partially due to the higher L/D ratios, the authors contend that it is more probably due to the higher surface irregularity and much higher flexibilities of paper and asbestos fibers, making strong fiber networks more probable.

Studies of polymer solutions and soap micelle solutions point to viscoelastic behavior as the fundamental key to turbulence modification (23). For fiber suspensions, fiber flexibility is linked to viscoelastic effects, for in a deforming field, the deformation state of the fiber depends on its time constant for bending and the history of force fields it has experienced.

Flexibility also affects the tendency to entangle and form a network, apart from the memory effect of single fibers. Network formation may be the key factor in determining fiber-turbulence interactions and is a history-dependent phenomenon. In any case, flexibility is expected to have a profound effect on drag reduction and other forms of fiber-turbulence interaction.

The network structure of the fibrous suspension is commonly and logically put forth as the key to drag reduction effects (2,17). Several factors which improve drag reduction (higher L/D, higher fiber flexibility, improved fiber dispersion) also directly affect network strength and flocculation. Degradation of the network is also correlated with a decrease in drag reducing ability. For example, Moyls and Sabersky (24) found that the loss of drag reducing ability in an asbestos suspension after exposure to high shear was reversed after a period of quiescence, suggesting that degradation occurred because of deformation and entanglement in the network rather than breakdown of the fibers themselves; the network apparently recovered some of its original drag-reducing properties.

One recent study showed drag reduction could be achieved by embedding small bundles of nylon fibers in the wall of a pipe (25). These strands did not appear to form traditional flocs, though the interaction of multiple fibers together may have simulated essential floc-like features.

Pirih and Swanson (26) investigated a suspension of milling yellow dye, which has submicron, rigid, rod-like crystals of aspect ratio 5.7, and is thus related to fibrous suspensions. Turbulent drag reduction on the order of 50% was observed for concentrations less than 0.5%. Turbulence was investigated with a constant-temperature anemometer. There was a decrease of 30-40% in longitudinal turbulence and an increase in length scales. Particles with larger relaxation times gave more extreme flow differences. No sudden transition from laminar to turbulent flow was apparent; the transition region was delayed and broadened by the suspension.

The mechanism of drag reduction in pulp suspensions is not well understood. Obviously, fibers, especially flocs of fibers, can dampen turbulence, but this alone is not sufficient to explain drag reduction. The fibers give the fluid a high apparent viscosity which allows more momentum transport to occur without Reynolds stresses, but the increased momentum transport means higher drag. When turbulence does occur, the same flocs and individual fibers which can act to dampen the turbulent eddies can act to efficiently transport momentum across large distances, again increasing the drag. How can fibers reduce turbulent momentum transfer without a corresponding increase in other forms of momentum transfer to give a net reduction in drag? The mechanisms of fiber-turbulence interactions and general particle-turbulence interactions must be explored to deal with this question.

Insights from Gas-Solid Turbulent Flows

Gore and Crowe (27) recently performed an analytical review of the literature to describe the conflicting conclusions dealing with particle flows in gas streams. Various researchers have worked with the addition of particles to turbulent flows and found the turbulence intensity to increase (eight references cited) and decrease (six references cited) from that of the carrier phase. Gore discovered that a critical parameter appears to be the ratio of particle diameter to a turbulent length scale (approximate length of the most energetic eddy in the pure carrier fluid, approximated as 0.2 times the pipe radius). When the ratio of particle diameter to turbulent length scale exceeds 0.1, turbulent intensity increases with particles present. For smaller particles, turbulent intensity is decreased. There was no reference made to cylindrical particles, but by using the length weighted fiber length to be the characteristic particle diameter, we may calculate a critical ratio of 0.092 for the present study – approximately on the Gore and Crowe demarcation line. However, the results from solid-gas flows are not directly applicable to solid-liquid flows. For instance, drag reduction with spherical particles is possible in gas but not in liquid flows.

The effect of spherical particles on turbulence in gas is related to the time scales for the particles and the turbulence. Defining the time scale for characteristic eddies as

$$T = \frac{D}{10\, u^*}, \tag{2}$$

where D is the pipe diameter and u^* is the turbulent friction velocity, $\sqrt{\tau_w/\rho_f}$, and using the Stokes particle time scale,

$$t_p = \frac{\rho_p}{18\, \rho_f} \frac{d^2}{\nu}, \tag{3}$$

Liljegren and Vlachos (28) obtain the ratio of particle to turbulent time scales:

$$\frac{t_p}{T} = \frac{5}{9} \frac{\rho_p}{\rho_f} \frac{u^* d}{\nu} \frac{d}{D}. \tag{4}$$

Liljegren and Vlachos studied particle-laden air flows in horizontal pipes, using 50 μm glass spheres with volume fractions of .0001 to 0.001. The particulate flow had a response time ratio of about 0.8, meaning the particles were small enough to respond to the largest scales of turbulence but were too large to respond completely to the most energetic (smaller) eddies. Their measurements suggested that the particles may enhance the largest scales and lead to additional production of turbulent energy. They find that turbulence in the pipe core flow can increase with particle loading, even though drag reduction is observed. This result is viewed as puzzling, and the authors call for "detailed measurements in the wall region ... to reconcile the observed drag reduction in the presence of increased particle velocity intensity in the pipe core...." They also note that Pfeffer and Rosetti (29), in their study of flows in vertical pipes, found particles with response time ratios up to about 8 cause drag reduction while simultaneously enhancing the turbulence in the core region of the pipe. The greatest degree of simultaneous drag reduction and turbulence enhancement was observed at a ratio near 1.6.

In reviewing past work for gas-solid flows in vertical pipes, where gravitational effects are unimportant, they find that turbulence appears to be first enhanced by the presence of very small particles but is suppressed by larger particles up to time ratios of about 22. Extremely large particles again enhance turbulence.

Liljegren and Vlachos also analyze the results of Boothroyd (30) in a study of diffusivity in the core of vertical gas flows with particles ranging from 0 to 40 μm. Diffusivity increased in a 3" pipe, but decreased in a 2" pipe. For 20 μm particles, the response time ratios ranged from 0.4 to 0.8 for the larger pipe and 0.9 to 1.8 for the smaller pipe. Liljegren and Vlachos conclude that diffusivity in the pipe core is enhanced by particles that are able to respond to the energetic eddies but is suppressed by larger particles.

Although some of the mechanisms of particle-turbulence interaction in gas flows will differ from fiber-turbulence interactions in liquid flows, the interesting finding from the gas flow studies is that particles can simultaneously cause drag reduction while inducing an increase in turbulent activity in the core of the pipe flow. Also of interest is the possibility of enhanced or decreased diffusivity, depending on the time scales.

Fiber Suspensions and Turbulence Structure

Relatively few measurements have been made of turbulent velocity fluctuations in turbulent flows of fibrous suspensions, and those that exist are usually of only one component, u', the longitudinal component. (The longitudinal, radial, and tangential rms turbulent velocity components are designated by u', v', and w', respectively, in this paper.)

Ek (31) made single-component LDV measurements in an 0.5% sulfate pulp suspension flowing through a 10 mm pipe. Measurements were based on scattering from fibers. Compared to water flow, Ek found that the longitudinal turbulence intensity of fiber fluctuations is increased near the wall ($1<r/R<0.5$) and is decreased near the center. No other velocity components were measured. (With fibers as long as 3 mm in the slurry, the pipe diameter is arguably too small to make measurements of industrial relevance. Indeed, friction measurements in pipes less than 75 mm in diameter may be subject to artifacts which limit their applicability to flows in larger pipes [32].)

McComb and Chan (33) made single-component laser-Doppler measurements in drag-reducing suspensions of asbestos fibers with extremely high l/d ratios (ca. 10^5) at 300 wppm. Tangential and longitudinal components were measured. At the lowest Reynolds number examined, Re = 1.4×10^4, 70% drag reduction was observed. The longitudinal fluctuating component, u', decreased compared to the fiber-free turbulent flow, but the tangential component, w', increased. Similar results were reported for Re = 9.0×10^3. This represents a profound modification of the turbulent structure, much more than simply a suppression of turbulence. (The decrease in u' is the opposite effect seen in drag-reducing polymer flows, where u' increases dramatically, and v' is decreased.) As the same solution was repeatedly passed through the flow system, there was a transition from "fibre-like" drag reduction to "polymer-like" drag reduction, with u' increasing and then decreasing back to that of the pure carrier fluid alone, and with drag reduction also decreasing. Examination of energy spectra, however, showed no evidence of a transition from one kind of drag reduction to another. Drag reduction magnitude also showed no obvious transition. These findings show that fibers can affect turbulence in complex ways.

At higher Reynolds numbers (3.2×10^4 and 5.3×10^4), u' was higher than in the carrier fluid alone even at high levels of drag reduction. Energy spectra showed a dip at length scales on the same order as the fiber length, suggesting resonant energy absorption by the fibers.

Recently, Steen (34) also has applied LDV to turbulence in a fibrous suspension. Steen used refractive index matching to make a transparent suspension of 10-μm diameter glass fibers in an alcohol solution. He examined consistencies of 1.2 and 12 g/l, fiber lengths of 1 and 3 mm, and Reynolds numbers of 8.5×10^3 and 6.5×10^4. Measurements were made in a vertical 23.8 mm pipe in developing flow following an orifice with 64% open area. The reported data show higher u' values at least near the wall when fibers are present, and in some cases u' exceeded that of a Newtonian fluid across the entire diameter. It is not known if drag reduction occurred in these flows, and the complex developing flow here makes application to other systems difficult.

A relevant study on turbulent structure dealt not with fibers but with a slurry of silica particles. Park et al. (35) made the first multi-component laser Doppler measurements in a transparent slurry. The slurry was 5.65 wt. % silica in Stoddard solvent with a matched index of refraction and behaved as a yield-power-law fluid, which bears some resemblance to the rheological properties of pulp suspensions. At low flow rates, a plug-like core was observed. In turbulent flow, the velocity profile resembled Newtonian turbulent flow. However, compared to a Newtonian fluid, the longitudinal turbulent intensity, u'/U, was slightly higher across almost the entire diameter, with a maximum increase of about 20% near the wall. The tangential turbulent intensity, w'/U, was much higher near the wall (.094 vs. .055 peak values) but was slightly lower for $0.1<y/D<0.9$.

Based on the above studies, the effect of fibers and particles in general on turbulence structure appears to be complex. Many authors have assumed that drag reduction in fiber suspensions is due to dampening of turbulence, especially radial velocity fluctuations, but this does not coincide with the the experimental data. For drag reduction to occur, Reynolds stresses must be decreased, and this need not require dampening of turbulence but rather a decoupling of radial and longitudinal velocity fluctuations. How fibers might induce this decoupling has not yet been explained, but a preliminary hypothesis will be advanced later in this paper.

Heat and Mass Transfer in Turbulent Fiber Suspensions

Relatively little has been done to investigate dispersion in turbulent fiber suspension flows. The most relevant work is that of Bobkowicz and Gauvin (36,37), who examined the dispersion of hot water injected at the centerline of a turbulent, aqueous suspension of non-flocculating nylon fibers in a 2" vertical tube. Dispersion was measured with a temperature probe, tracking the spread of the heated water across the diameter of the pipe. They found that radial dispersion significantly increased in the presence of fibers, even when drag reduction occurred. Fiber aspect ratios ranged from 11.7 to 51.1, and fiber concentrations ranged from 0.5 to 6.0%. For the lowest aspect ratios, the pressure loss curve was essentially the same as that for pure water, but at the highest aspect ratios, substantial drag reduction was observed. There seemed to be no dependence on fiber diameter or length alone but only on the aspect ratio, l/d. An examination of their data suggests that dispersion enhancement did not behave in the same way as drag reduction, for at the lowest l/d values, where no drag reduction was apparent, enhanced dispersion was still evident.

The results of Bobkowicz and Gauvin do not correspond to the findings of Moyls and Sabersky (24), who found heat transfer from the wall was decreased in dilute, turbulent suspensions of asbestos fibers. Drag reduction was observed, but the decrease in heat transfer was even greater than the decrease in momentum transfer.

A visual study of dye dispersion in pulp flow was conducted by Andersson (38), who injected dye at the centerline of flow through a tall, narrow rectangular channel. A grid was placed at the inlet of the channel to create isotropic turbulence. Pulp concentrations of 1.1-4.7 g/l were used. Andersson observed decreased dispersion of the dye compared to dispersion in water flow; dispersion decreased with increasing concentration.

Some authors have briefly mentioned observations from dye injection experiments in pulp flows. Luthi (39), for example, injected dye into the centerline of a 6% pulp suspension flow that was in a drag-reducing regime. This high-consistency slurry still had a pronounced plug flow region in the core; as a result, the dye stream was still intact and undispersed in the core after 1.5 m of flow. When dye was injected at the wall, however, it quickly dispersed "sideways," presumably in the tangential direction, indicating that active turbulence was present in an annular region. Duffy et al. (2) also performed dye injection at the wall in pipe flow to determine that turbulence in the outer annulus occurs at flow rates below the drag reduction regime. A fibrous plug in the core of the flow can still exist well into the drag reduction zone.

Other directly applicable studies of turbulent dispersion in fibrous flows have not been found in the literature.

EXPERIMENTAL

Approach

The experimental approach for examining flow loop dispersion properties was divided into the following steps:

1. Prepare a model pulp system and characterize its components.
2. Modify an existing flow loop so that turbulent dispersion rates may be measured and controlled, and stock sampling may be made.
3. Design a sensor/data acquisition system for measuring dispersion rates in turbulent flow, on-line.
4. Acquire data from dispersion experiments of saline solution into pure water flows and into dilute pulp suspension flows.

Materials

The pulp for this study was a bleached kraft hardwood donated by Mead Corporation (the Escanaba, Michigan mill) and obtained in dry lap form. A species identification revealed the presence of mostly Populus spp. The original pulp was tested for fiber length using the Kajaani FS-100; this pulp reportedly contained a weight weighted average fiber length of 0.90 mm with 17.4 arithmetic percent fines. A reduction of the fines fraction, for adsorption experiments, was facilitated using a pilot paper machine. A fine tissue was made on the machine so as to reduce retention. The fines were successfully reduced to 4.4 percent in the tissue. The tissue was stored at approximately 30 percent solids in refrigeration (40°C) with one percent formaldehyde solution until needed for flow loop experiments.

A model filler material consisting of a 0.40 μm diameter polystyrene latex spheres, donated by Morton Thiokol, was added to the pulp solution for subsequent adsorption and electrophoretic mobility measurements. The injected material was a commercial polyacrylamide (PAM) retention aid (Allied Colloids, Percol 175) of high molecular weight ($9x10^6$) and medium charge density (10 mole percent). The polymer was used as a tracer for the efficiency of pipeline mixing. An 0.05 percent solution of the polymer was prepared in 0.05 M KCl; the conductivity of the polymer solution was higher than that of the pipeline upstream flow and could be tracked by an on-line conductivity probe. The probe tracked only the presence of the saline solution as the KCl attachment onto surfaces was negligible and the removal of the PAM from the saline solution had no effect on the measured conductivity.

Equipment

An existing flow loop was modified to include a testing section with a polymer preparation station and sampling and sewer loops. An operating schematic is shown in Figure 2. The flow loop consisted of a stock dilution tank (2500 gal.) for dispersing the fiber and latex. Flow from the dilution tank maintained a constant level in a mix tank (960 gal.). The mix tank was fitted with steam and cold water pipes so that a constant temperature of 23°C (73°F) could be maintained. PVC piping, 7.62 cm inner diameter (3" Schedule 40), was fitted throughout the flow loop. A variable speed pump on the bottom of the mix tank maintained a constant flow rate between 0.6 and 5.5 m/s (2 and 18 fps). A magnetic flowmeter (Rosemount) with a digital output was used to control and maintain flow rates.

A 3.05 m (10 ft) pipe test section, installed 78 pipe diameters from an elbow, consisted of a polymer injection port and seven "hot-tap" mountings. The fittings connecting the test section piping were smooth routed to reduce disruptions to the flow pattern. Flow could continue through a recycling loop and back into the mix tank. During experiments involving an injection of material through the test section port, air-actuated ball valves (Dezurik) were manually switched to close off the recycling loop and open the sampling/sewer section. A computer-controlled, air-actuated, V-port ball valve using a PID (proportional-integral-derivative) algorithm controller (written in Turbo Pascal) was used to stabilize and maintain a set-point flow rate. Sampling and sewer legs of the loop were also controlled by air-actuated manually-controlled ball valves.

To quantify dispersion, a conductivity probe was sequentially placed in seven hot-tap mountings along the pipeline test section downstream from the injection port. A prototype probe and meter were built by TBI-Bailey Controls and modified by IPST electronics personnel to output data at 1000 Hz. Data were acquired through an A/D interface board to a 286-processor computer. The conductivity probe was 1.5 cm (19/32") in diameter, and thus may introduce some perturbation to the flow field. The degree of perturbation, however, should be the same for both water and dilute pulp suspensions and should not affect trends in measured dispersion of the KCl solution.

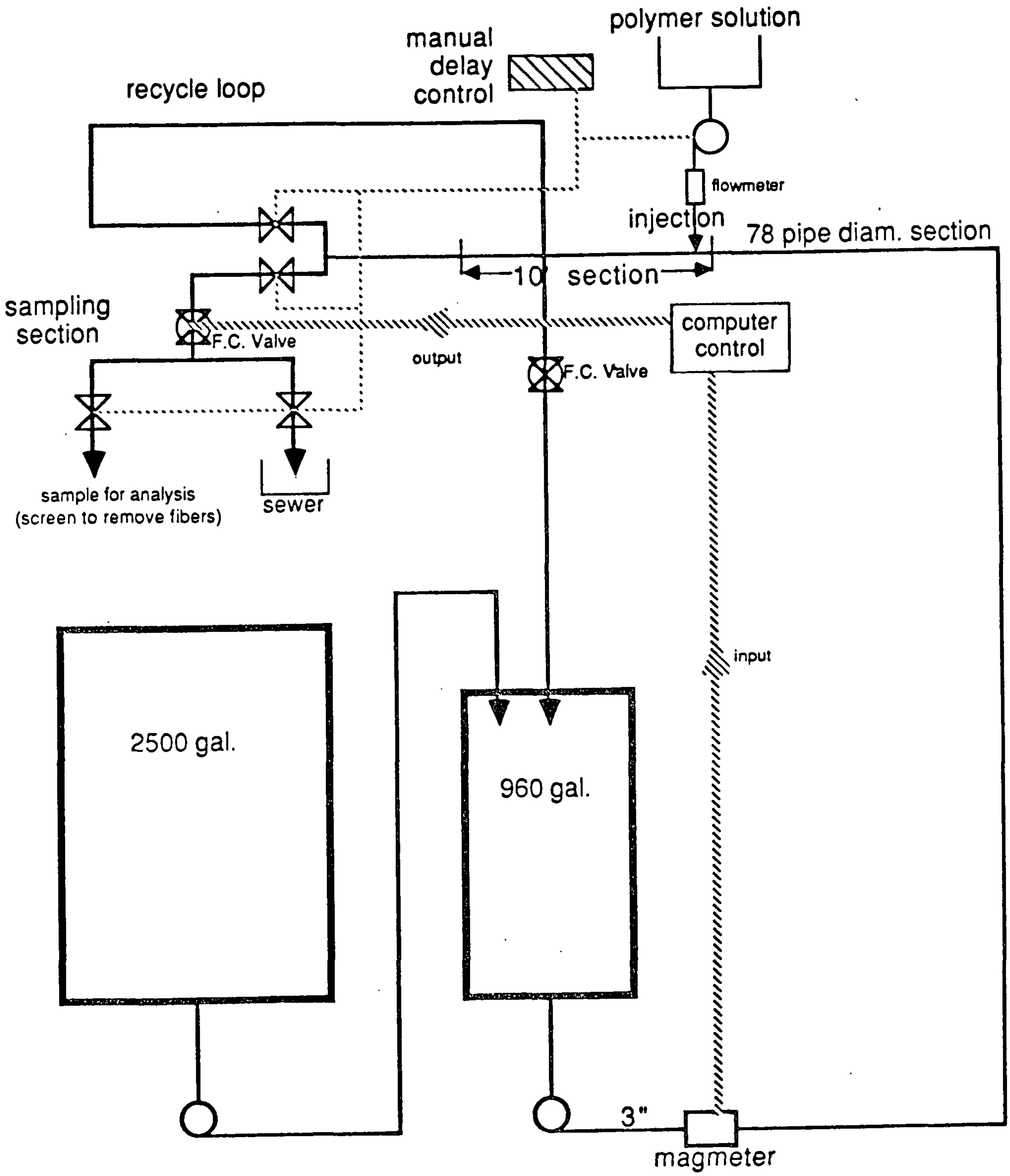

Figure 2. Flow loop used in this study.

Flow and Injection Conditions

A variety of velocity conditions were studied. Three injection port sizes and four pipe velocities were used. All test conditions are summarized in Table 1 below. Numbers in parentheses after code letters for each run indicate that pulp flows as well as water flows were studied at that condition; the number is the series number of the pulp flow run.

Code	Inj. Port Size, cm	Pipe Vel., m/s	Jet vel., m/s	Vel. ratio	Pipe Re	Jet Re
A(1)	0.635	0.91	0.69	0.752	63,100	1,550
B	0.635	1.83	1.38	0.752	126,100	3,120
C	0.635	2.74	2.06	0.752	189,200	4,690
D(2)	0.635	4.57	3.44	0.752	315,300	7,780
E(3)	0.125	0.91	2.75	3.007	63,100	3,120
F	0.3175	1.83	5.50	3.007	126,100	6,230
G	0.3175	2.74	8.25	3.007	189,200	9,330
H(4)	0.3175	4.57	13.75	3.007	315,300	15,570
I(5)	0.0625	0.91	11.00	12.03	63,100	6,230
J(7)	0.3175	0.91	5.50	6.013	63,100	6,230
K(6)	0.635	0.91	1.38	1.503	63,100	3,120
L(8)	0.635	0.91	2.06	2.252	63,100	4,670
M(9)	0.3175	0.91	8.25	9.02	63,100	9,340

Table 1. Pipe flow and injection jet conditions used in the present study.

We must now consider the state of turbulence for the flows in this study. The velocity for onset to turbulence is a function of several factors. These include pulp consistency, fiber morphology, and pipe diameter. For the present study, a hardwood fiber at 0.5 percent consistency (%C) was examined in a 7.6-cm (3") inner diameter pipe at Reynolds numbers that varied between $6x10^4$ and $3.2x10^5$ (pipe velocities of 0.9 to 4.6 m/s or 3 to 15 ft/s). Robertson and Mason (40) studied fiber flows in a 2.2 cm (7/8") ID glass tube and found a transition to turbulence above 60 cm/s (2 ft/s or a Reynolds number of $1.3x10^4$). In a comparison of softwood and hardwood chemical pulps, they found the flocculating tendency to be greatly reduced for the shorter, stiffer hardwood pulps. It would thus appear reasonable to conclude that the flow conditions examined in the present study were turbulent, although friction factor data were not acquired.

Other studies can be compared to the present experiments to assess the flow state. Experimental data of low consistency fiber flows in 2" pipes have been presented by Daily and Bugliarello (4,5). Between consistencies of 0.50 and 1.00 percent, the transition to turbulence could range in Reynolds number from $1x10^4$ to $9x10^4$, depending on the pulp type. For suspensions of increased length to diameter ratio and flexibility, there were increased departures from Newtonian behavior. Long-fibered, flexible pulps such as a southern pine bleached kraft had extended transition regimes. Short-fibered pulps that were less flexible such as poplar groundwood (fiber length 0.49 mm) had abrupt transitions to turbulence at a relatively low Reynolds number, becoming fully turbulent at about $4x10^4$.

Mih and Parker (3) obtained friction factor data for hardwood kraft; characteristic curves were found for a 0.5%C pulp suspension in 2" and 4" pipes. The point of maximum drag reduction was found at Reynolds numbers of $8x10^4$ and $1.5x10^5$, and the point of turn toward the water line at $2x10^5$ and $5x10^5$ for the 2" and 4" pipes, respectively.

Moller and Duffy (41) present empirical equations to describe the inflection points for the friction loss curve. The minimum in the friction curve, the onset of drag reduction, and the maximum drag reduction equations were each given; these values for the present study were calculated as 0.12, 0.46 and 1.0 m/s, respectively. The minimum velocity examined (.91 m/s) is thus expected to be near the point of maximum drag reduction.

DATA

Conductivity Contour Plots

Conductivity profiles were measured across the pipe diameter at three positions downstream of the injection port corresponding to x/D of 1, 3, and 5. The profiles were made in the same plane that contained the center of the injected plane. The conductivity data were converted to contour plots using the Spyglass Transform™ software package for Apple Macintosh II computers. The raw data were read into an array of 16 columns and 200 rows, with column and row location corresponding to physical location of the conductivity measurement. Missing cells were filled with linear interpolation first for columns, then for rows, and the entire array was then smoothed with 12 smoothing passes. An interpolated color image and a line contour plot were then generated from the array. Because of the small amount of data available, the contours presented here cannot be taken as exact; indeed, artifacts from the data filling and smoothing procedures are likely to be present. However, the contours do capture the essence of the data and are helpful in graphically portraying major differences.

The reproducibility of the conductivity measurements can be examined by comparing three replicate measurements for water flow in Figures 3-5. Some scatter is evident, but the basic concentration patterns are similar. Figures 6 and 7 also show similar reproducibility for measurements in pulp flow.

Comparison of dispersion in pulp flow to water flow at the same velocity ratio and Reynolds numbers is possible by examining Figures 6, 7, and 8. The saline solution appears to be dispersing more rapidly in the pulp flow than in the water flow. The spread of the plume in the radial direction does not appear to be significantly increased, but the concentrations are declining more rapidly with distance. The same behavior can be seen in Figures 9-12, which compare pulp and water flow for flow case D . This effect was observed in all data sets collected.

Results for higher velocity ratios are shown in Figures 13-16. Again, the dispersion appears to be higher in the pulp flow.

The jet velocity ratio controls the degree of jet penetration into the pipe flow. A ratio of 12 almost takes the jet to the far wall of the pipe, and a ratio of 3 brings the jet centerline to about one-half pipe radius from the near wall.

The conductivity results show no evidence of plug flow in the 0.5% hardwood slurry. At high injection velocities, the injected jet appears to pass through the core to the other wall with no unusual artifacts that would be created by a coherent plug of fibers.

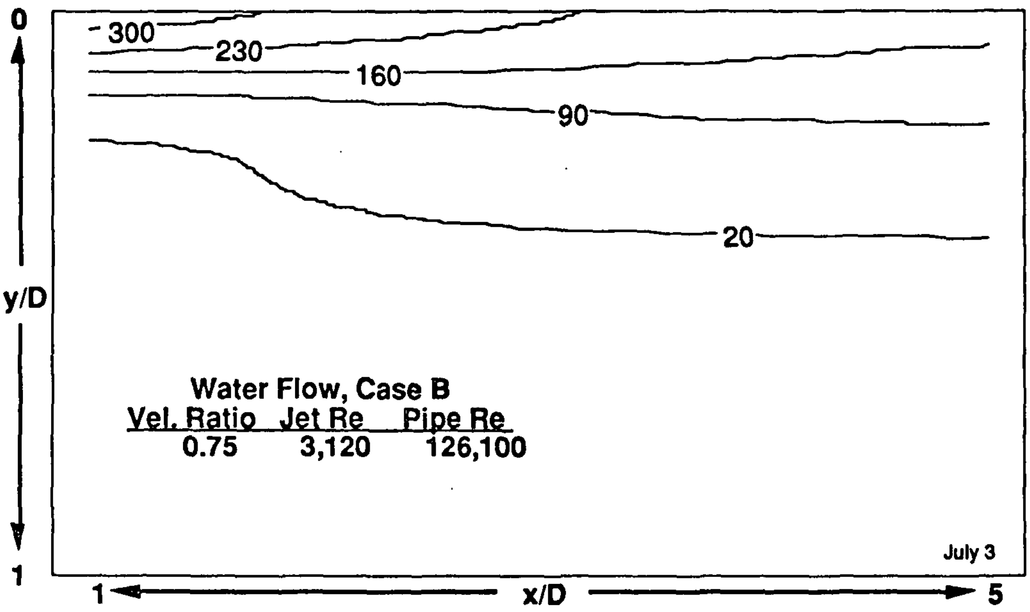

Figure 3. Dispersion in water flow. Case B, run 1.

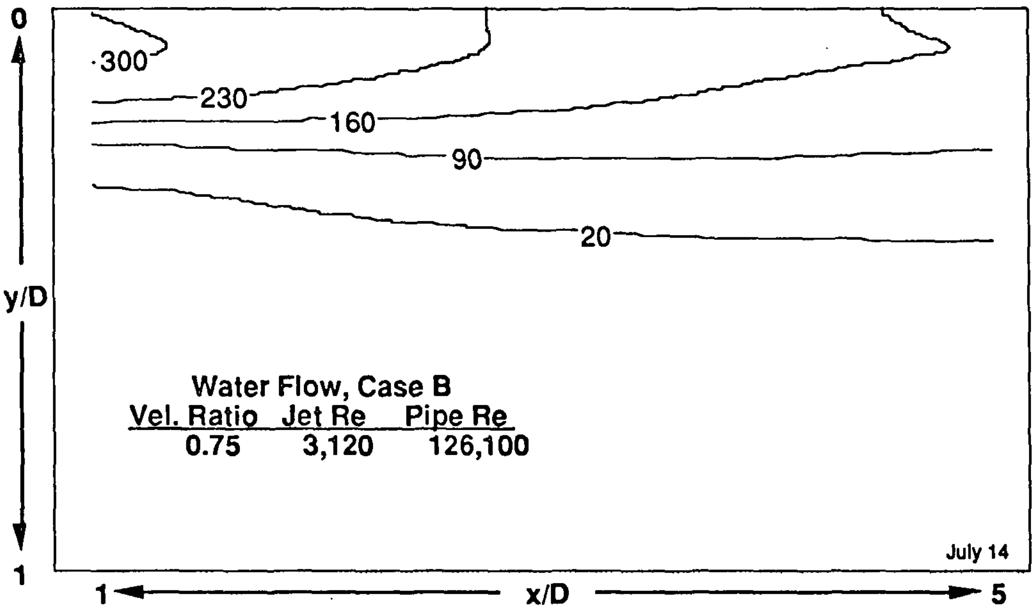

Figure 4. Dispersion in water flow. Case B, run 2.

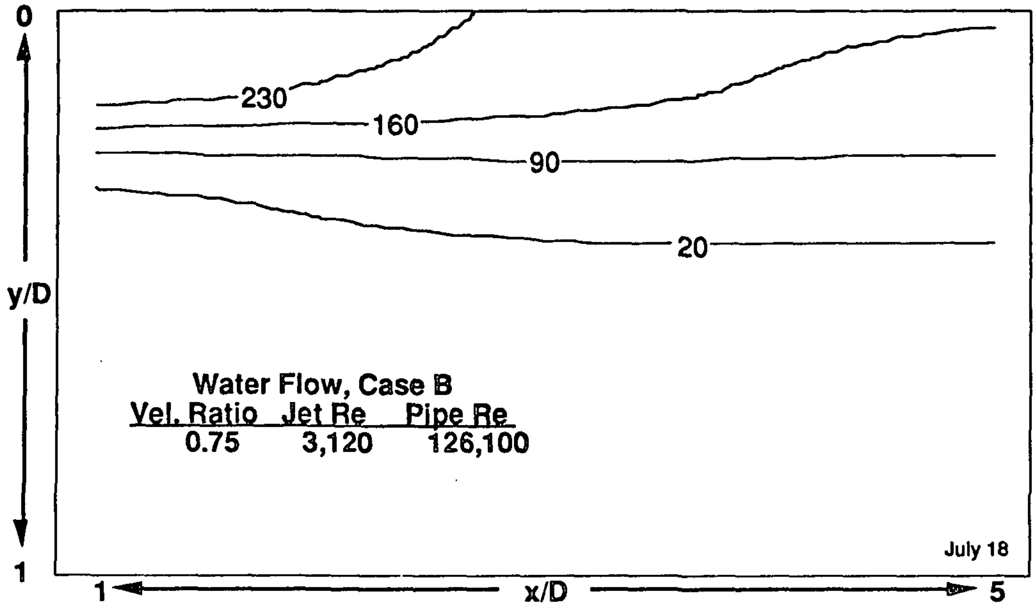

Figure 5. Dispersion in water flow. Case B, run 3.

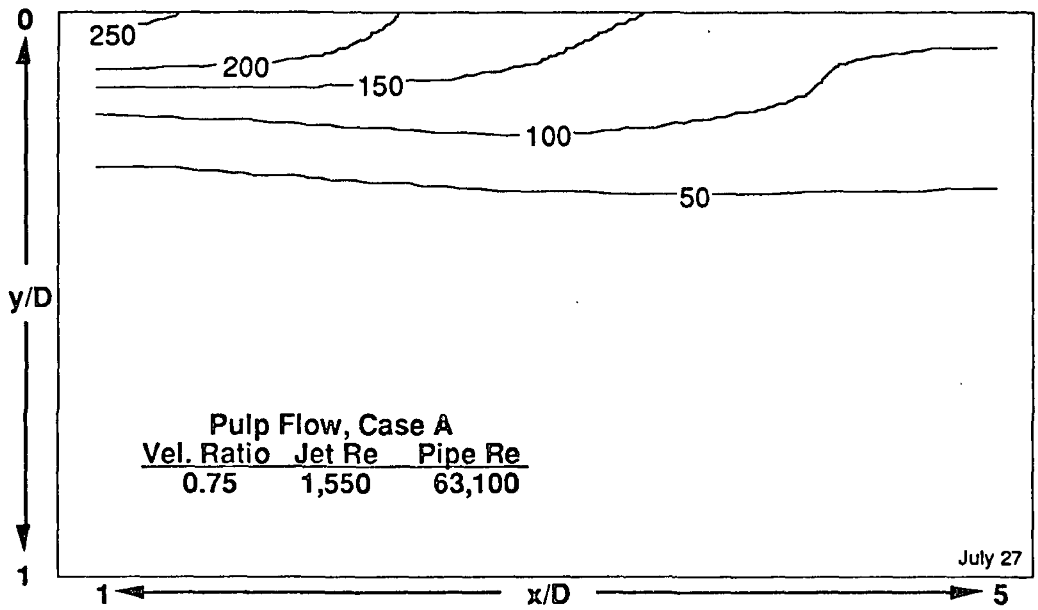

Figure 6. Dispersion in pulp suspension flow. Case A, run 1.

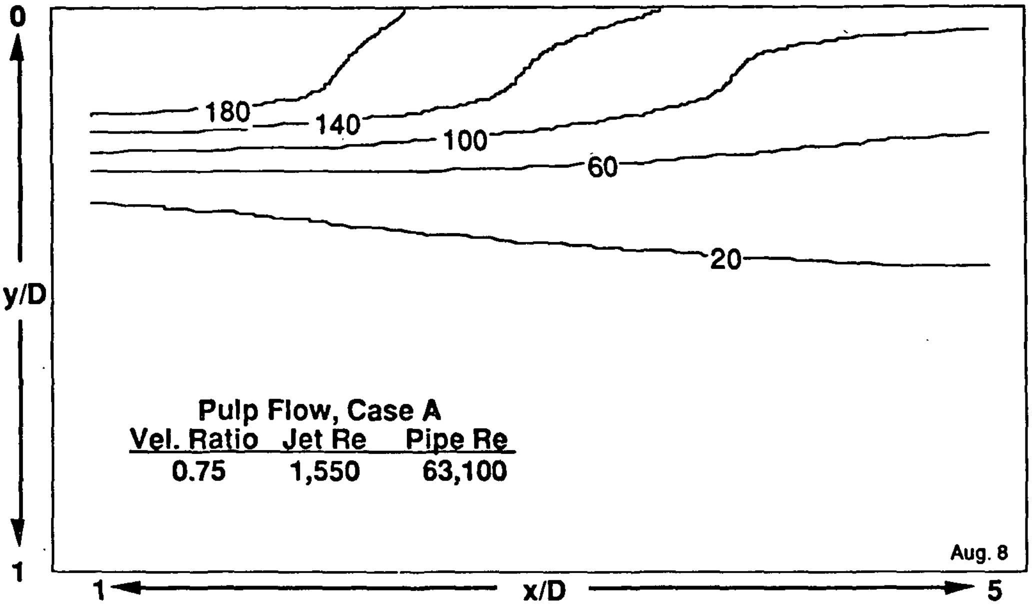

Figure 7. Dispersion in pulp suspension flow. Case A, run 2.

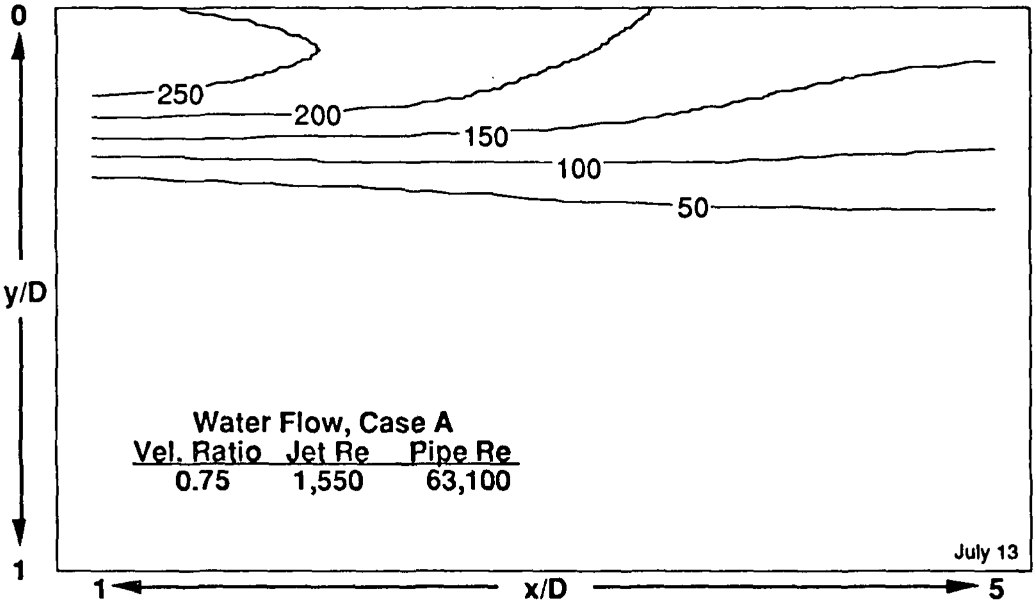

Figure 8. Dispersion in water flow. Case A.

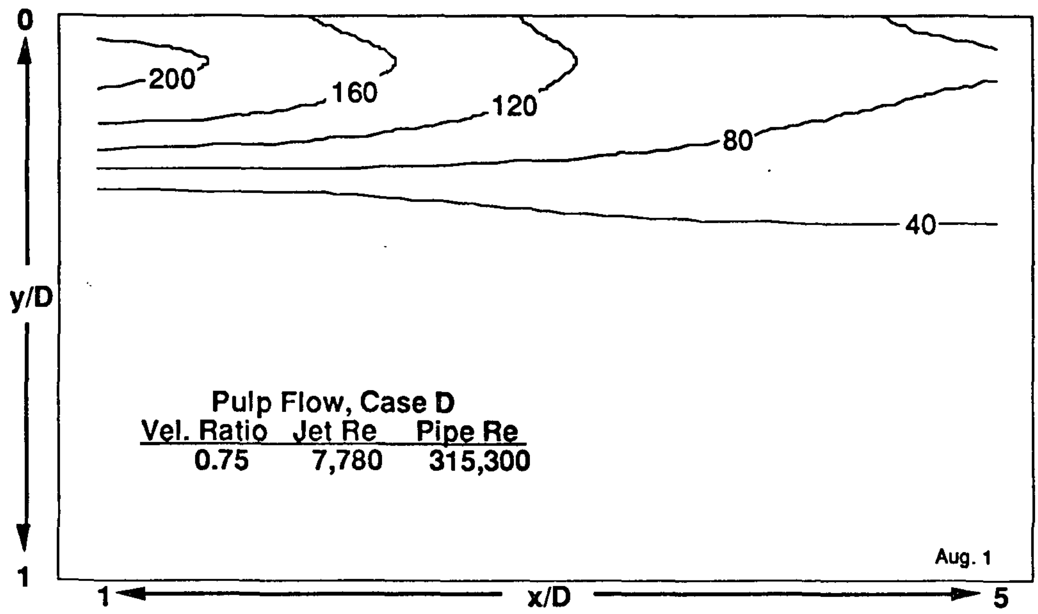

Figure 9. Dispersion in pulp suspension flow. Case D, run 1.

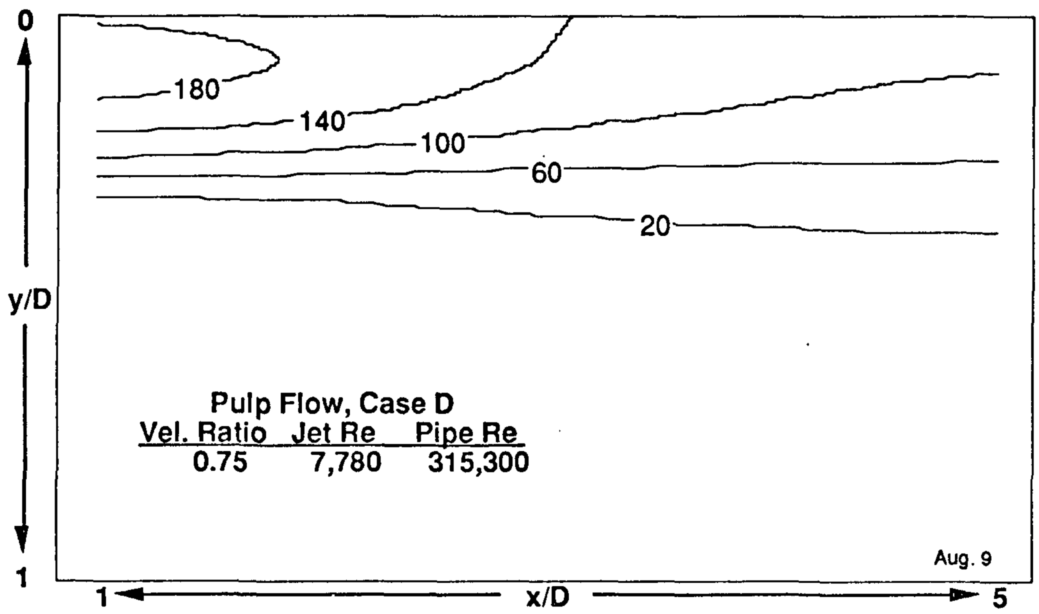

Figure 10. Dispersion in pulp suspension flow. Case D, run 2.

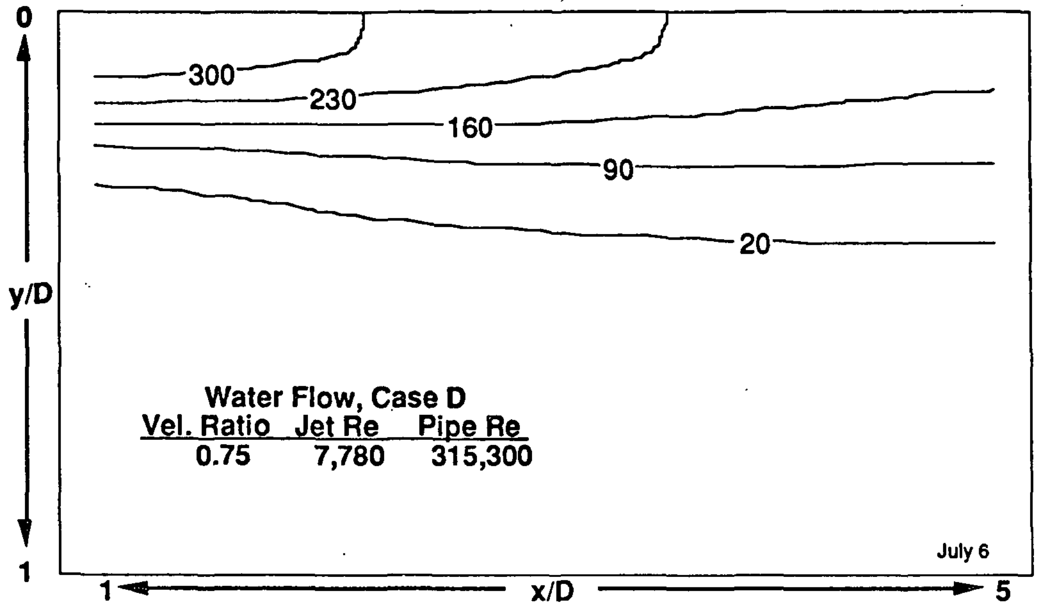

Figure 11. Dispersion in water flow. Case D, run 1.

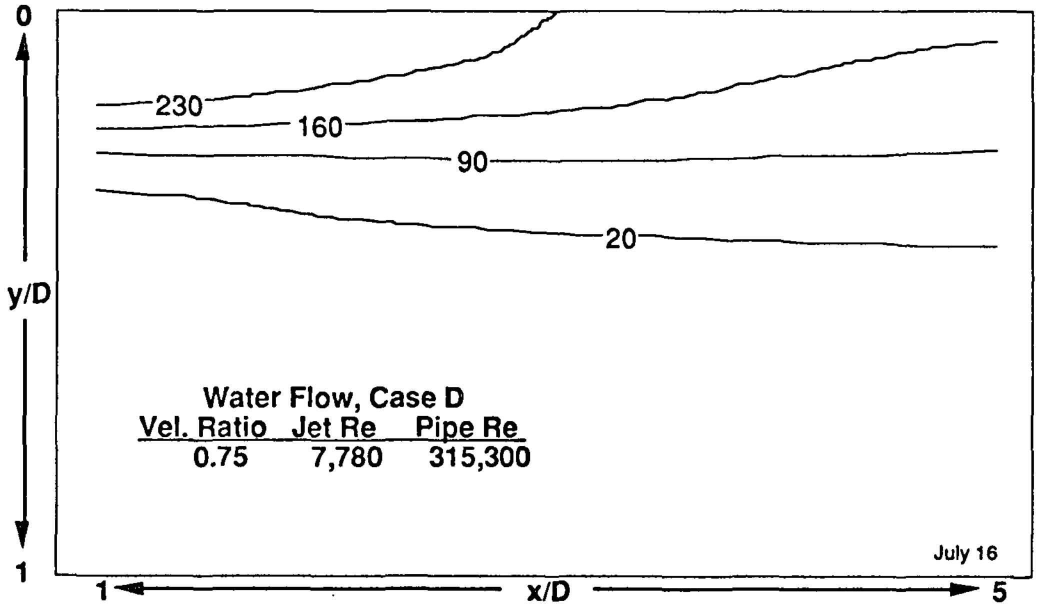

Figure 12. Dispersion in water flow. Case D, run 2.

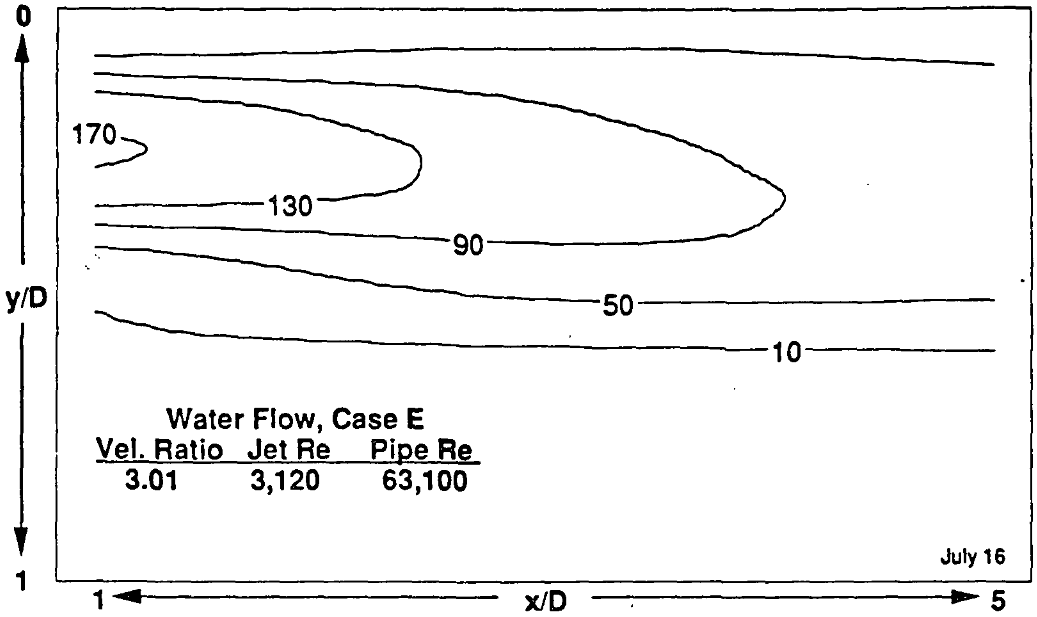

Figure 13. Dispersion in water flow. Case E.

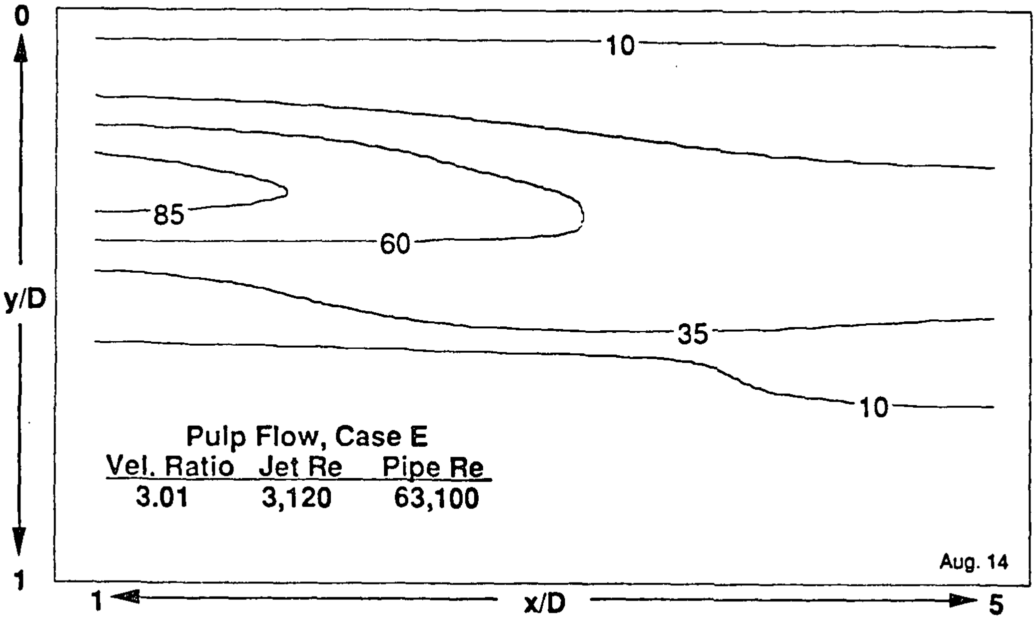

Figure 14. Dispersion in pulp flow. Case E.

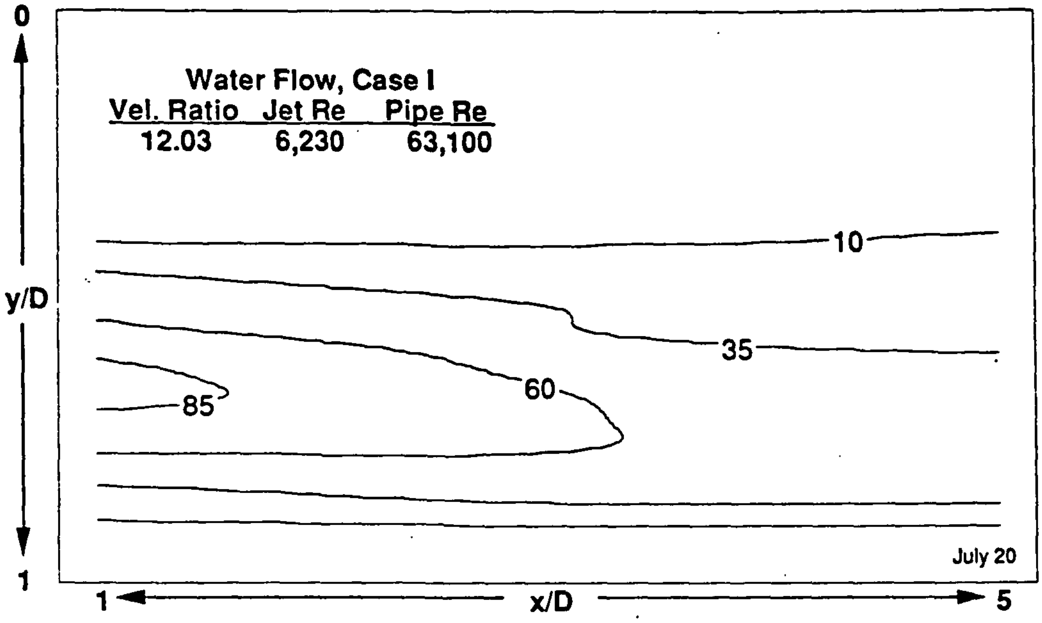

Figure 15. Dispersion in water flow. Case I.

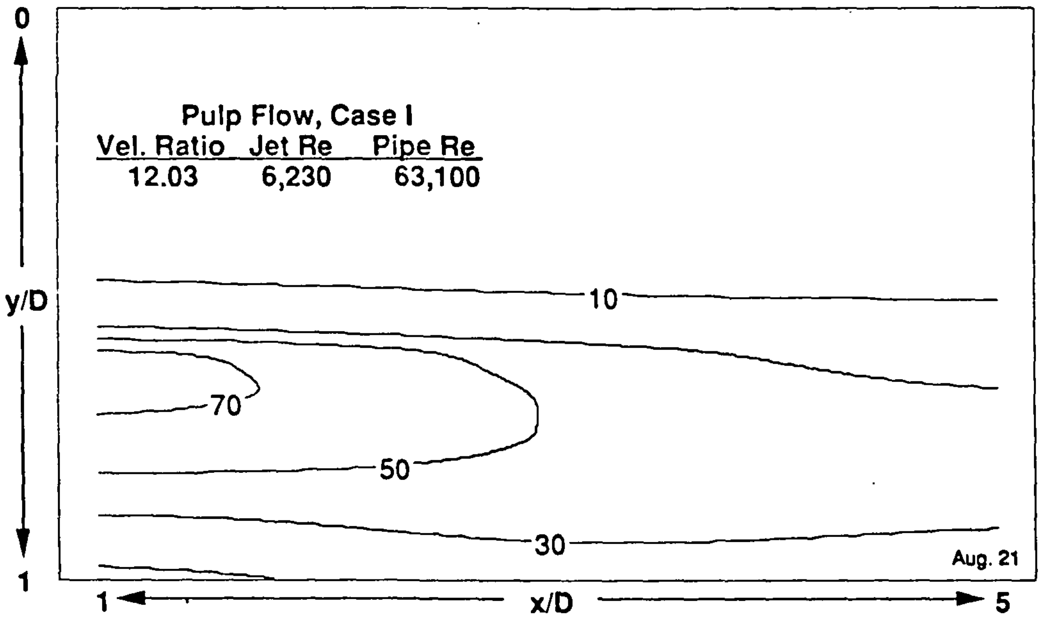

Figure 16. Dispersion in pulp flow. Case I.

DISCUSSION OF DISPERSION RESULTS

Limitations in the Data

Unfortunately, the experiments of this study were not designed with fiber-turbulence interactions in mind but were designed to study polymer absorption as a function of mixing conditions. As a result, when we apply the results to fundamental issues of turbulent flow, the data and experimental technqies can only be taken as preliminary. Further work using more appropriate techniques and better instrumentation is now being planned.

In addition to the small data set, several complications exist in the experiments which limit their value. For example, the probe was intrusive, though equally so in both water and pulp. Mass transfer may also have been affected by the polymer in the injected jet as well as the turbulent structure of the pipe flow, for the polymer itself is a drag-reducing material at high Reynolds number. However, the effect of the polymer in the injection jet on the main pipe flow is expected to be essentially the same for both the water and pulp suspension flows. The time for significant polymer absorption to occur on the fibers is much longer than the dwell time within the test section where conductivity measurements were made. As a result, even if the polymer was affecting the turbulence, we do not expect polymer absorption on fibers to cause a significant difference in the suspension flow compared to the pure water flow. However, future experiments will avoid the undesired complexity introduced by the presence of polymer in the injection stream.

Implications for Turbulent Flow

Although pressure-drop measurements were not made for the flow of the fibrous suspension, flows of fibrous suspensions at similar concentrations and Reynolds numbers are known to be drag reducing in turbulent flow (3), as discussed above. Hardwood slurries, in particular, at similar concentrations are reported to be in the drag reducing turbulent regime at Reynolds numbers below and equal to the Reynolds number of the flow studied here. We will therefore assume that these measurements were made in a drag-reducing regime.

The observation of higher turbulent dispersion in the fibrous suspension than in pure water seems counter-intuitive for a drag-reducing flow. Drag reduction implies a decrease in turbulent momentum transfer, and a decrease in momentum transfer intuitively should be linked to a decrease in mass transfer. In fact, even if the flow were at lower Reynolds numbers where drag enhancement occurs, increased dispersion would not be expected. Drag enhancement at low flow rates is due to the higher apparent viscosity of the slurry that increases viscous momentum transfer but dampens turbulence and turbulent dispersion.

We are then faced with a puzzle: how is it possible for fibers to increase turbulent dispersion while decreasing turbulent momentum transfer? To answer this question, we must first understand how fibers modify the structure of turbulence.

The Effect of Fibers on Turbulent Flows

Effect on Dispersion

As mentioned above, the work of Bobkowicz and Gauvin (36,37) showed that radial dispersion in a turbulent pipe flow significantly increased in the presence of nylon fibers even when drag reduction occurred. Fiber aspect ratios ranged from 11.7 to 51.1, and fiber concentrations ranged from 0.5 to 6.0%. There seemed to be no dependence on fiber diameter or length alone, but only on the aspect ratio, l/d. An examination of their data suggests that dispersion enhancement did not behave in the same way as drag reduction, for at the lowest l/d values, where no drag reduction was apparent, enhanced dispersion was still evident. The authors point out that the work by Daily and Bugliarello (4,5) measured turbulence intensity decreases in the longitudinal direction and suggest that an increase in radial turbulence is not necessarily inconsistent with decreased longitudinal turbulence.

Later, this work received criticisms as being inconsistent with generally accepted beliefs of fiber turbulence damping. Possible reasons reported were that the flow was not fully turbulent and that there may have been particle interference with the sensing probe. We find no such grounds for criticism; the flow conditions were probably turbulent given the reported parameters and the evidence of dispersion. Also, particle interference does not explain a larger dispersion area measured by the thermistor probe.

The observation that fiber length alone does not affect drag reduction suggests floc size *per se* is not the primary factor in reducing drag. The elastic absorption and redistribution of turbulent energy by bending fibers may be the key factor. We will later take up the hypothesis that dispersion enhancement may be more directly related to floc size than is drag reduction, emphasizing the possibility that the mechanisms of drag reduction and dispersion enhancement need not be the same. Measurements of w' and u' for flows of various fiber lengths but identical l/d values would be useful in resolving this issue.

Above we noted the heat transfer study of Moyls and Sabersky (42), who found heat transfer from the wall was decreased in drag-reducing suspensions of asbestos fibers. One would then expect mass transfer to be reduced in this system. The difference may imply that mass transfer at the wall was reduced and became a limiting factor in this study; mass and heat transfer in the core could still have been elevated. For the data presented in the current study, the injected fluid penetrated into the turbulent core to various degrees, so a reduction in transport at the wall may have been present but undetected. Further research is needed to explore this point. Along these lines, Vaseleski and Metzner (17) speculate that drag-reducing fibrous suspension flows may, in some cases, have enhanced heat and mass transfer at the wall. They cautiously advance this speculation based on predicted turbulent velocity profiles which show the intersection of the core and the viscous sublayer moving closer together as fiber concentration is increased. Extrapolated measurements of velocity profiles by Lee and Duffy (43), however, show that the intersection point is unchanged in flows of pulp suspensions.

We also noted Andersson's observation (38) that dye dispersion was decreased by the presence of fibers in flow through a narrow rectangular channel after a grid. However, the decaying turbulent structure in the post-grid rectangular channel is not expected to resemble the turbulence in pipe flow. The Andersson study does support the intuitive perception that fibers can dampen turbulence and decrease dispersion.

The possibility of increased dispersion in drag-reducing flows of fiber suspensions stands in contrast to results for drag-reducing flows of polymeric solutions (11). In these polymer flows, a significant decrease in turbulent dispersion has been observed (12,13,44).

Effect on Velocity Fluctuations

Above, we discussed several previous measurements of turbulence velocities in fiber suspensions. Applying these findings to our study, it is significant that McComb and Chan (33) observed an increase in tangential rms velocity in a strongly drag-reducing fibrous flow at a Reynolds number of 1.4×10^4. An elevated w' would explain the observed effect of fibers on turbulent dispersion in this study. We wish to emphasize McComb's and Chan's finding that turbulence structure, in terms of relative magnitudes of individual fluctuating components, may undergo dramatic transitions that are not paralleled in the drag reduction and energy spectra results. Drag reduction is a manifestation of turbulent stresses, reflecting the correlation between individual components. It may thus be that the mechanism which affects the individual rms components does not strongly affect the correlations between them.

Limited Value of Velocity Profiles

Examination of velocity is often used to deduce information about the turbulent mechanisms of a flow. Based on an examination of velocity profiles, others have concluded that the mechanisms of turbulence production in fiber suspensions are the same as those in Newtonian fluids. However, the results of Park et al. (35) discussed above show that examination of the velocity profile alone may not reveal important changes in the turbulent structure. In their flow, the turbulent velocity profile was virtually identical to that observed in Newtonian fluids, but the turbulence structure (and hence the mechanisms of turbulent production) showed significant changes. Fiber-turbulence interactions require further information to be understood.

Mechanisms for Fiber-Turbulence Interactions

Here we tentatively discuss several mechanisms by which a fiber suspension can modify the turbulence to achieve observed effects.

The Critical Issue of Reynolds Stress

Reynolds stresses are of fundamental importance in understanding the structure of turbulence but are often overlooked in discussion of turbulence modification by fibers. For example, in explaining their finding of enhanced dispersion in a drag-reducing flow, Bobkowicz and Gauvin (36,37) suggest that inter-particle collisions of fibers aligned in the flow direction could increase the radial turbulence (v') relative to the longitudinal turbulence. They then incorrectly argue that drag reduction can be accompanied by an increase in v' as long as u' decreases more. A decrease in turbulent momentum transfer, however, means that the Reynolds stress term, $\overline{u\,v}$, has decreased. The Reynolds stress represents the correlation between the fluctuating components u and v. The Reynolds stress can be positive, negative, or zero, depending on how u' and v' are correlated, independent of the magnitude of either rms component (45).

Discussions of drag reduction in pulp suspensions typically propose dampened turbulence fluctuations, especially dampened radial fluctuations, as the mechanism for reduced shear (2,3). Certainly, if v' drops to zero, then the Reynolds stress will also become zero and purely viscous transport will result. In general, however, a decrease in v' alone need not decrease drag. Indeed, it is not clear that v' actually is decreased, for measurements of this component are rare. There is evidence, however, that u' can decrease with a simultaneous increase in w', and in some drag reducing flows, just the opposite has been seen (33). Though the mechanisms are different, a similar scenario occurs in polymeric drag reducing flows, where strong increases in u' are accompanied by decreases in v'. Virk (11) notes that a *decoupling* of axial and radial turbulent velocities, rather than a suppression of turbulent intensity, causes drag reduction in polymer solutions. It appears that the same conclusion should be drawn for fibrous suspensions.

The Role of Individual Fibers

The region in which Reynolds stresses are suppressed appears to be further from the wall for fibrous suspensions than in polymeric drag reducing flows. In the latter case, the suppressed Reynolds stresses occur in a narrow region near the wall, corresponding roughly to the buffer region, where polymeric molecules can be elongated sufficiently to uncoil and cause a large increase in apparent viscosity, damping the Reynolds-stress-containing eddies there (15). In fibrous suspensions, the mechanism of drag reduction appears to occur outside the buffer layer in the turbulent core. A possible explanation for the suppression of Reynolds stresses is the great resistance of fiber suspensions to the elongational deformations (46) caused by some energy-containing eddies entering the core. Elongational forces may be dampened, and possibly even converted into vorticity, causing fibers and flocs to rotate in the shear field. This transport mechanism imposes a more laminar characteristic to the flow, similar to the transport in the viscous sublayer, where vorticity is equal to the strain rate and significant elongational forces can not occur (14,15).

Spectral data indicate energy absorption occurs at a length scale on the order of fiber length (33). Free fibers or fibers loosely bound in flocs can absorb and redistribute energy from eddies at this length scale in any part of the flow at any Reynolds number. However, the effect that this absorption has on the turbulence structure depends on the relation of fiber length to the local scales of turbulence. Drag reduction results show a maximum degree of drag reduction occurs at a specific Reynolds number. Increasing the Reynolds number further reduces the degree of drag reduction, and in some cases a return to the friction factor of the pure solvent is reported, although some degree of drag reduction usually persists. As Re increases, length scales decrease, and the fibers are less effective in restructuring the most energetic eddies to induce drag reduction turbulence.

The Role of Flocs and Networks

Rather than considering the fibrous suspension as a single network, a system of separate but interacting, coherent flocs is a more realistic model, especially at low consistencies. The presence of a floc in the flow locally increases the scale of motion. A burst of turbulent energy can affect fibers far away through forces conveyed by the mesh in the flow. Likewise, a moving floc can act as a mechanism for mass transfer as it rolls and sweeps fluid around its periphery.

The motion of flocs in the flow may also provide a mechanism for increasing longitudinal and tangential velocity fluctuations while suppressing radial velocity fluctuations. Consider a coherent floc which extends into low-velocity regions near the wall. The floc will be carried forward at a velocity greater than the velocity near the wall. As the flow sweeps through the lower-velocity fluid near the wall, its primary effect may be to displace fluid to the side, something like the hull of a boat forcing water to the side; some fluid will also be swept forward. As the floc passes, fluid rushes back to fill the void. The presence of the wall restricts radial motion, so the dominant effect of the displacements caused by the floc will be to increase tangential and longitudinal fluctuations. This heuristic model is intended merely to illustrate how flocs might contribute to the change in turbulent structure which some authors have observed; its validity is open to question.

The gradient in fiber concentration near the wall may also contribute to the decreased Reynolds stress: a burst of low-fiber concentration fluid from the wall may be impeded in the radial direction as it enters a high-fiber concentration zone, and some of the kinetic energy of the burst may be deflected in the tangential or longitudinal directions. The process may be analogous to the anisotropy induced by a free surface in otherwise isotropic turbulence: vertical turbulent scales near the surface are "blocked" and redistributed into lateral components of turbulence (47). The presence of flocs may also serve to stericly redirect turbulent fluctuations, causing a restructuring of turbulence. A more detailed discussion of these theories will be presented in a subsequent paper.

SUMMARY

The intuitive view of turbulence damping in the drag reducing flows of fibrous suspensions is an inadequate explanation for the observed changes in turbulence structure. A profound indication of an altered turbulence structure is the observation that increased turbulent dispersion and decreased momentum transfer can occur simultaneously when fibers are added to a turbulent water flow. The presence of flocs and entangled fibers in a flow may force a redirection of turbulent bursts which could alter the structure of turbulence. The resistance to elongation of fluid elements with fibers may also contribute to the changed turbulence structure. These findings and theories, however, are preliminary. Further work is needed to examine these mechanisms more accurately.

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